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Wave propagation of micro-polar fluid used as blood in a Elastic tube

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ABSTRACT
Presented herein are the theoretical analysis of wave propagation of micro-polar fluid used as blood in a elastic tube. It is clear that the micro rotational velocity damping factor of the micro-polar fluid in the elastic tube. The analysis is restricted to the tube with in walls and wave whose amplitude is infinitely small and wave length large compared to the radius of the tube.

KEYWORDS: Micro-polar fluids, stress, strain, damping factor, radial constant.

I. INTRODUCTION

Physiologist concerned the blood circulation problem of wave propagation in a system in which the distensibility of the tube is of far greater importance than the compressibility of the fluid. The problem too has been studied by various authors theoretically as well as experimentally. Some investigators[3,5] have studied the problem of pressure waves in a elastic tube.

Micro-polar fluid introduced by Eringen[2] are proving worthwhile from various points of view. These fluid are capable of sustaining stress moments and body forces and are influenced by spin inertia. Much work has been done to study the behavior of such types of fluids. Renuka[8] studied some flow problems of micro-polar fluid between two parallel plates and in a circular cylinder. Willson[11,12], investigated the stability of the flow of these fluid, down on inclined plane. He also gave analysis on the basic flow of these fluids.

Laxmana Rao[6] gave the general solution of the field equation of micro-polar fluids. These fluids are capable of explaining the micro-rotation effects of small particles in a small volume element. Thus, the flow properties of the fluids with certain additives as well as flow behavior of blood cells can be studies with the help of theory. Thus, an attempted for studying the unsteady flow of these problems for micro- polar fluid will be very useful for engineering and biological systems.

Muller's[7] experiment indicated that the wave velocity increases considerably with increasing frequency of disturbance. Womersley[13] treated the problem of the pulsatile flow in a elastic unconstrained tube. Womersley[15] contains the tube longitudinally. Atbeck and Lew[1] adds to this effect of anisotrophy and longitudinal, radial constraints. Skalak[9] considered the role of wave propagation in blood flow and found the result. Karreman[4] following White more[10] attempt to extend the latters analysis. Yadav A.K. & Pokhriyal S.C.[15] considered the wave propagation in micro-polar fluid in a flexible tube. Yadav A.K. & Kumar S.[14] investigated load capacity of paid bearing.

The purpose of this paper is to investigate the effect wave propagation in a simple manner and showing clearly all physical assumptions.

II. FORMULATION OF THE PROBLEM

Equation of the motion under the assumptions stated above in cylindrical coordinate system and that the velocity induced in a fluid is very small enables us to ignore non-linear term reduces to ;

$$\frac{\partial^2 u}{\partial z^2} - \frac{\rho}{(\mu_\vartheta + k_\vartheta)}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right] - \frac{\partial^2 w}{\partial z \partial r} - \frac{k_\vartheta}{(\mu_\vartheta + k_\vartheta)}\frac{\partial v_\theta}{\partial z} + \frac{\partial p}{\partial r} = 0 \qquad (1)$$

$$(\mu_v + k_\vartheta)\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \rho\left\{\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right\} - \frac{\partial^2 u}{\partial z \partial r} - \frac{1}{r}\frac{\partial u}{\partial z}\right] + k_\vartheta\left(\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\right) - \frac{\partial p}{\partial z} = 0 \qquad (2)$$

$$\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} + \frac{\partial^2 v_\theta}{\partial z^2} - \frac{2k_v v_\theta}{\gamma_\vartheta} - \frac{v_\theta}{r^2} - \frac{\rho J}{\gamma_\vartheta}\left[\frac{\partial v_\theta}{\partial t} + u\frac{\partial v_\theta}{\partial r} + w\frac{\partial v_\theta}{\partial z}\right] + \frac{k_\vartheta}{\gamma_\vartheta}\frac{\partial u}{\partial z} = 0 \qquad (3)$$

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We assume that the velocities have small variation in azimuthal way and along the tube. The equation of motion reduce to

$$\rho\frac{\partial\omega}{\partial t} = -\frac{1}{(\mu_v+k_\vartheta)}\frac{\partial p}{\partial z} + \frac{\partial^2\omega}{\partial r^2} + \frac{1}{r}\frac{\partial\omega}{\partial r} + \frac{k_\vartheta}{(\mu_\vartheta+k_\vartheta)}\left[\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\right] = 0 \quad (4)$$

and

$$\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} - \frac{\rho J}{\gamma_\vartheta}\frac{\partial v_\theta}{\partial t} - \frac{2k_\vartheta}{\gamma_\vartheta}v_\theta = 0 \quad (5)$$

Equation of continuity. Is

$$\frac{1}{r}\frac{\partial(ur)}{\partial r} + \frac{\partial\omega}{\partial z} = 0 \quad (6)$$

Normal compressive stress is given by

$$E_{rr} = p - 2\mu_v\frac{\partial u}{\partial r} \quad (7)$$

The shearing stress acting in the direction parallel to the axis of the tube on an element of area perpendicular to a radius is given by

$$E_{rz} = \mu_v\left(\frac{\partial\omega}{\partial r} + \frac{\partial u}{\partial z}\right) \quad (8)$$

III. LONG HARMONIC WAVE

We wish to investigate the propagation of forced pressure waves which are harmonic in time and all the variable can be expressed as follows

$$\left.\begin{array}{ll} \xi = Se^{i(kz-\omega t)}, & \eta = \Delta e^{i(kz-\omega t)} \\ u = w(r)e^{i(kz-\omega t)}, & w = u(r)e^{i(kz-\omega t)} \\ p = Pe^{i(kz-\omega t)}, & V_\theta = V(r)e^{i(kz-\omega t)} \end{array}\right\} \quad (9)$$

Where ω is circular frequency of forced disturbance and $k = k_1 + ik_2$

Where k_1 = wave number

k_2 = decay of the disturbance

For small domping $(modulusk) = k_1 = \frac{1}{\lambda}$

Where $2\pi\lambda$ is wave length of disturbance

Now substituting expression (9) in equation (4) and (5) we get

$$-i\omega\rho u = -\frac{ikP}{(k_\vartheta+\mu_\vartheta)} + \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] + \frac{k_\vartheta}{(\mu_\vartheta+k_\vartheta)}\left[\frac{\partial v}{\partial r} + \frac{v}{r}\right] \quad (10)$$

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \left[\frac{i\omega J\rho}{\gamma_\vartheta} - \frac{1}{r^2} - \frac{2k_\vartheta}{\gamma_\vartheta}\right]v = 0 \quad (11)$$

Equation of continuity reduces to

$$\frac{1}{r}\frac{d}{dr}(r\omega) + iku = 0 \quad (12)$$

Re-ranging Eq. (11) and (12),we get

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + i\omega\rho u + \frac{k_\vartheta}{(\mu_\vartheta+k_\vartheta)}\left[\frac{\partial v}{\partial r} + \frac{v}{r}\right] = \frac{ikP}{(\mu_\vartheta+k_\vartheta)} \quad (13)$$

$$r^2\frac{\partial^2 v}{\partial r^2} + r\frac{\partial v}{\partial r} + (\alpha_1^2 r^2 - 1)v = 0 \quad (14)$$

Where $\alpha_1^2 = \frac{Ji\omega\rho - 2k_\vartheta}{r_\vartheta}$ (15)

And

$$\frac{1}{r}\frac{d}{dr}(\omega r + ik_\vartheta) = 0 \quad (16)$$

The boundery condition at the wall are the velocity component of the fluid be equal to those of walls , we have

$$u = u(R_0)e^{i(kx-\omega t)} \quad (17)$$

$$\omega = \omega(R_0)e^{i(kx-\omega t)} \quad (18)$$

Solving equation (13) and (14) using the boundary condition ;

$$u = AJ_0(\alpha r) + \frac{k_\vartheta}{(\mu_\vartheta+k_\vartheta)}\left[AJ_0(\alpha_1 r) + AJ_1\frac{(\alpha_1)r}{r}\right] + \frac{\mu_\vartheta\sigma}{(\mu_\vartheta+k_\vartheta)}\frac{du}{dr} \quad \text{at } r = R_0/ \quad (19)$$

$$\vartheta = A_1 J_1(\alpha_1 r) \quad (20)$$

Where $\alpha^2 = i\omega\rho$ (21)

$$\alpha_1^2 = \frac{\omega\rho Ji - 2k_\vartheta}{r_\vartheta} \quad (22)$$

and

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$$\left.\frac{du}{dr}\right|_{R=R_0} = -\alpha A J_1(\alpha R_0) - \alpha_1 A_1 J_1(\alpha_1 R_0) \tag{23}$$

Then

$$u(r) = AJ_0(\alpha r) + \frac{k_\vartheta}{(\mu_\vartheta + k_\vartheta)}[A_1 J_0(\alpha_1 r) + A_1 J_1(\alpha_1 r)/r] - \frac{i\sigma\mu_\vartheta}{(\mu_\vartheta + k_\vartheta)}[\alpha A J_1(\alpha R_0) + \alpha A_1 J_1(\alpha_1 R_0)] \tag{24}$$

From equation (12), we get

$$w = -\frac{ik}{r}\int_0^r urdr \tag{25}$$

The lower limit on the integrate have been chosen to satisfy the equation (20), (23) & (25) and integrating, we get

$$w(r) = -\frac{iAk}{\alpha}J_1(\alpha r) - \frac{ikk_\vartheta}{(\mu_\vartheta + k_\vartheta)}AJ_1(\alpha_1 r) + \frac{i\sigma r\mu_\vartheta}{2(\mu_\vartheta + k_\vartheta)}[\alpha A J_1(\alpha R_0) + \alpha_1 A J_1(\alpha_1 R_0)] \tag{26}$$

Let Q_n be the volumetric flow rate for n^{th} harmonic,

$$Q_n = \int_0^a 2\pi r\omega(r)\, dr$$

$$= 2\pi a\left[-\frac{iAk}{\alpha}J_2(\alpha a)\right] - \frac{ikk_\vartheta}{(\mu_\vartheta + k_\vartheta)}A_1 J_2(\alpha_1 a) + \frac{i\sigma\mu_\vartheta a}{2(\mu_\vartheta + k_\vartheta)}[\alpha A J_2(\alpha R_0) + \alpha_1 A_1 J_2(\alpha_1 R_0)] \tag{27}$$

IV. RESULTS

The analysis of this paper may be extended is a opinions the stress strain relation for the tube material involved additional time derivatives of either stress or strain. It is clear that the second time derivatives of ε_{ik} will affect the leading term of the expression for the velocity.

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